Title:
Development of the Prototype of a Mechanical Arm with a Semicircular Path via the Isometric Transformations of a Tetrahedron

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## Introduction:

This project starts from the premise of creating a mechanical arm with a semicircular path via the isometric transformations of rotation and translation of a tetrahedron, because in the isometric transformation, the geometric shapes does not change its shape or size only changes its position in space (orientation and direction), leaving, the original shape and the final shape geometrically similar and congruent.

Of the five Platonic solids, the tetrahedron is the most feasible regular polygon to recreate the movements of a mechanical arm, because a tetrahedron is a regular polyhedron with congruent faces of regular polygons that meet at a vertex, and when there is an intersection between two inclined tetrahedrons, the cutting plane of the truncated tetrahedron remains parallel to the base of the tetrahedron that cuts, and connections can be placed perpendicular to the base of the truncated tetrahedrons to connect them, therefore, it is possible to follow a semi-circular path with a sequence of tetrahedrons.

In this paper we present the geometric process that was carried out to create the 3D prototype of a mechanical arm, using the following methodology:

- Analysis of the movement of a mechanical arm
- Study of isometric transformations (translation and rotation) of a tetrahedron
- Study of an semicircular path to create intersections and unions of the tetrahedrons
- Generation of the modules, junctions and spherical articulations
- Analysis of movement through virtual simulations
- 3D printing of the components
- Assembly of the prototype


## Main Idea:

A mechanical arm is an articulated arm that has similarity to the anatomy and movements of the human arm for their anthropomorphic characteristics of waist, shoulder, arm, elbow, wrist and hand which generally have four axes of motion (Fig. 1(a)).

If we analyze the motion of a mechanical arm (Fig. 1(b)) we can see that the direction of its axis is a straight line (Fig. 1(c)), but in the study that was conducted in the laboratory of the university, we realized that the direction of the axis of a mechanical arm can also be a curved line, i.e. a semicircular path (Fig. 1(d)).


Fig. 1: Mechanical arm (1) Similar to the anatomy of the human arm, (b) Movement of rotation, (c) Straight line direction and (d) Curve line direction.

In the design of the mechanical arm we begin with the trace of the projections of two tetrahedrons, whose measure of the edges is 113.1371 mm , with a distance of 150 mm between its centers, the base of one of the tetrahedron is down and the base of the other tetrahedron upward, then in the vertical plane of the orthogonal projection is traced the semicircular path from the vertex of the tetrahedron \# 1 till the base center of the tetrahedron \# 2 (Fig. 2(a)).

In the vertical plane of the orthogonal projection the tetrahedrons are copied and moved it upward from the supporting point, point " b ", till the midpoint of the height of the tetrahedron, then the tetrahedrons are rotated from the supporting point, until the upper point, point "d", of the height of the tetrahedron (the vertex) intersects with the semicircular path (Fig. 2 (a) and (b)).


Fig. 2: Tetrahedrons (a) Semicircular path and isometric transformation and (b)) Projections, axis and nomenclature.

This procedure is repeated as many times as necessary until one of the tetrahedrons intersects with the tetrahedron \# 2, therefore and due to the distance of the semicircular path between tetrahedrons, the mechanical arm will have ten truncated tetrahedrons that intersect each other (Fig. 3).


Fig. 3: Mechanical arm.

So there will be two types of intersections, the first, will be a complete intersection, wherein the cutting plane of each tetrahedron is determined (Fig. 4(a)) and the second is an incomplete intersection, wherein two tetrahedrons are joined to form a single shape (Fig. 4(b)).


Fig. 4: Intersection (a) Complete and (b) Incomplete.
To assembling and recreate the movements of the mechanical arm, our design has two spherical articulations, eight tetrahedrons having one or two holes and seven tetrahedrons having a connection cylinder, all of them positioned perpendicular to the inclined planes of the tetrahedrons (Fig. 5).


Fig. 5: Junctions (a) One hole and one cylinder, (b) Two holes and (c) Spherical articulation.
The spherical articulations are placed within a rectangular base and within the tetrahedron \# 2, so that the two pieces are cut into equal parts and subsequently assembled by means of cylinders. (Fig. 6 (a) and (b)).


Fig. 6: Spherical articulation (a) Within a rectangular base and (b) Within the tetrahedrons that are joined.

Once finished with the modeling of all parts, the mechanical arm is assembled virtually, to simulate the circular motion (Fig. 7 (a, b, c, d, e and f)), and then are printed all parts to assemble the prototype to test it (Fig. 8 (a, b, c, d and e)).


Fig. 7: Mechanical arm: (a, b, c, d, e and f) Simulation of the circular motion.


Fig. 8: Mechanical arm: (a, b, c, d and e) Prototype with the circular motion.
In the prototype which is moved manually, it can be observed (Fig. (8) and (9)) that the mechanical arm has two axes of motion: shoulder and wrist, that rotates at the starting point from $0^{\circ}$ to $38^{\circ}$ to the right and to the left rotates till that the vertex of tetrahedron \# 2 reaches the ground; rotates at the end point from $0^{\circ}$ to $12^{\circ}$, and in both cases rotate $360^{\circ}$.


Fig. 9 Mechanical arm: Render with the circular motion.
The continuity, stability and balance in the prototype is achieved, as can be seen in Figure \# 8, because, the movements of translation and rotation were performed from the height and center of the tetrahedrons when the tetrahedrons had already been displaced and rotated, in the semi-circular path each tetrahedron have different angle of inclination, and the cutting planes do not intersect at the same point, since in eight of the truncated tetrahedrons, the edges of the base and the edge "ab" has the same magnitudes, and five of the edges does not have the same dimensions (Fig. 10 and 11), (Tab.1)).

## Conclusions:

The results show that it is possible to recreate the movement of a mechanical arm using two spherical articulations and a series of truncated tetrahedrons following a semi-circular pattern, due to the isometric transformations of the tetrahedrons, and that the connections are placed perpendicular to the base of the truncated tetrahedron to connect a sequence of tetrahedrons, because when performing isometric transformation into the tetrahedrons, what changes is the angle of inclination of
the tetrahedrons in space, so this research contributes to conceptualize the isometric transformations of the geometric shapes to create models that require movement.


Fig. 10: Front view of truncated tetrahedrons with hole and cylinders: (a) Conceptual view and (b) Orthogonal projection.


Fig. 11: Tetrahedron: (a) Orthogonal projection and (b) Edges.

| Tetrahedrons | Base <br> edges | Edges upper base |  |  | Inclined edges |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | each | a | b | c | d | e | f |
| 2 | 56.5685 | 25.9884 | 32.4726 | 33.6235 | 28.2843 | 19.393 | 33.7434 |
| 4 | 56.5685 | 26.1975 | 3.3942 | 32.6106 | 28.2843 | 20.8985 | 33.1361 |
| 6 | 56.5685 | 26.3017 | 30.9567 | 32.1841 | 28.2843 | 21.5502 | 32.8461 |
| 8 | 56.5685 | 26.3539 | 30.75 .87 | 31.9868 | 28.2843 | 21.8556 | 32.7039 |
| 10 | 56.5685 | 26.38 | 30.6644 | 31.8918 | 28.2843 | 22.0037 | 32.6334 |
| 12 | 56.5685 | 26.393 | 30.6184 | 31.8452 | 28.2843 | 22.0766 | 32.5983 |
| 14 | 56.5685 | 26.3996 | 30.5957 | 31.822 | 28.2843 | 22.129 | 32.5808 |
| 16 | 56.5685 | 28.7847 | 33.3435 | 34.6803 | 28.2843 | 19.0241 | 30.4072 |

Tab. 1: Dimension of the edges of the truncated tetrahedrons.

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