Title:
A New Approach for Irregular Porous Structure Modeling based on Centroidal Voronoi Tessellation and B-Spline

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Introduction:

Porous structure is a particular type of solid with pores. Different pore parameters, such as size, shape, porosity, and pore interconnection, lead to diverse porous structures. Compared to conventional solids, porous structures possess many superior properties including lower density, lower thermal conductivity, lower stiffness, lower strength, larger compressive strain, and larger surface areas. In recent years, porous structures have been widely utilized into the areas of tissue engineering, energy absorbers, thermal insulators, and lightweight structures.

The wide applications of porous structures have attracted a lot of attention on porous structure modeling. Computer-Aided Design (CAD) plays an important role on this research topic because of the easy manipulation and user-friendly interactions of CAD packages, as well as the advanced Additive Manufacturing (AM) technologies that transform complex CAD models into valid products. Among various theories and approaches for porous structure modeling in CAD, the unit cell method [2-4], [9] is one of the most broadly adopted. Armed with a library of primitive cell structures and automatic assembly strategies [3], [4], unit cell methods provide a powerful tool for both 2D and 3D porous structure modelling. However, these methods are hampered by their poor capacities to exhibit pore diversity in both size and morphology. Alternative to unit cell methods, a set of reconstruction methods have been proposed to mimic the porous structure in nature or construct irregular porous structures [8], [13]. The advantage of these reconstruction methods is that diverse pore shapes with non-uniform distribution can be generated. However, these methods require existing porous models must be at hand, which significantly restricts their applicability and efficacy.

To solve the aforementioned problems, Kou and Tan [12] proposed a simple but effective geometric representation for irregular porous structures based on stochastic Voronoi tessellation (VT). The key advantage of their method is that it generates irregular porous artifacts and no exiting media (e.g. MRI-based models) are called for. This method was further extended to represent irregular porous structures with graded pore sizes and distributions [11]. Despite that their methods enjoy much efficacy in constructing irregular porous structures on a global level; these methods lack the abilities to precisely control the local pore size and distribution since the Voronoi generators are randomly distributed during the process of porous structure modeling. Another disadvantage of their methods is that the obtained porous structures may not be effectively analyzed by using either conventional finite element methods (e.g. finite element methods relying on triangular or tetrahedral meshes) or Voronoi cell finite element methods (VCFEMs) [7], [14]. By using the former ones, a large number of elements are always needed to accurately formulate the physical behavior of porous structures, leading to tremendous computational costs [1], [16]; by using the latter ones, ill-conditioned stiffness matrices and simulation errors will be introduced because of the overly large or small angles within each Voronoi cell of porous structures [15].

Motivated to tackle these issues, we propose a new approach for irregular porous structure modeling based on centroidal Voronoi tessellation (CVT) and B-Spline. Compared to existing methods for porous structure modeling based on VT or Quadtree (Octree) [10], our approach are much more flexible to control the pore size and distribution both globally and locally. In addition, our method can effectively prevent overly large or small angles within Voronoi cells, which are the source of simulation errors and computational burdens [15].

Main Idea:

The proposed method for irregular porous structure modeling mainly includes two steps: domain partition by CVT, and porous structure generation.

Domain partition by CVT

Centroidal Voronoi tessellations (CVTs) are Voronoi tessellations (VTs) of a region that their generating points coincide with the centroids of the corresponding Voronoi regions [5]. In the literature, VTs and CVTs have been greatly utilized to partition geometric domains and generate polygonal meshes [6], [17].

Given an open bounded domain $\Omega \in \Re^d$ and a set of distinct points $\{\mathbf{x}_i\}_{i=1}^n$ belong to Ω, for each point $\mathbf{x}_i$, the corresponding Voronoi region V_i consists of all the points in Ω that are closer to $\mathbf{x}_i$ than to any other points in the set, i.e.:

$$V_i = \left\{ \mathbf{x} \in \Omega \,|\, \left|\mathbf{x} - \mathbf{x}_i\right| < \left|\mathbf{x} - \mathbf{x}_j\right| \;\; \text{for } j = 1,\ldots,n \text{ and } j \neq i \right\} \tag{1}$$

where $|\cdot|$ denotes the Euclidean distance in $\Re^d$. We refer to $\{V_i\}_{i=1}^n$ as the VT of Ω and the points $\{\mathbf{x}_i\}_{i=1}^n$ as the associated generating points.

Given a density function $\rho(\mathbf{x})$ which is positive on Ω, for any region $V \subset \Omega$, we define $\mathbf{x}^*$, the centroid of V by

$$\mathbf{x}^* = \frac{\int_V \mathbf{x}\rho(\mathbf{x})d\mathbf{x}}{\int_V \rho(\mathbf{x})d\mathbf{x}} \tag{2}$$

A VT of Ω, $\left\{(\mathbf{x}_i,\, V_i)\right\}_{i=1}^n$, is called as CVT if and only if the generating points $\{\mathbf{x}_i\}_{i=1}^n$ associated with the Voronoi regions $\{V_i\}_{i=1}^n$ are also the centroids of those regions, i.e., if and only if $\mathbf{x}_i = \mathbf{x}_i^*$ for $i = 1, \ldots, n$. Fig. 1. shows a VT and a CVT bounded by an arbitrary curve with respect to a uniform density function $\rho(\mathbf{x}) = 1$.

Porous structure generation

After constructing the CVT over the domain of interest, the porous structure is then generated by adding a pore into each Voronoi cell. The generation process of each pore in our method is similar to that in conventional methods based on VT [11], [12], except that we use CVT instead of VT. For the sake of simplicity, only a brief introduction of the contruction process of the porous structure is given in what follows. Interesting readers may refer to [11], [12] for a compresensive study.

Fig. 2. demonstrates the procedure how a pore is built in a single Voronoi cell. Give an arbitrary Voronoi cell V_1 as shown in Fig. 2. (a), a scaled Voronoi cell is first generated according to the expression as below:

$$\mathbf{z}_{si} = \mathbf{z}_c + t\left(\mathbf{z}_i - \mathbf{z}_c\right),\; 0 < t \leq 1 \tag{3}$$

where $\mathbf{z}_i$ denotes the ith vertex in V_1, $\mathbf{z}_{si}$ denotes the corresponding vertex in the scaled Voronoi cell V_2, $\mathbf{z}_c$ denotes the centroid of V_1, and t is the scaling factor which is utilized to control the size of the scaled Voronoi cell. A B-Spline curve S is then generated with the vertices of V_2 as the control points.

Boolean difference is finally conducted between the Voronoi cell, V_1, and the area enclosed by the B-Spline curve, *S*, generating a porous Voronoi cell (see Fig. 2. (b)).

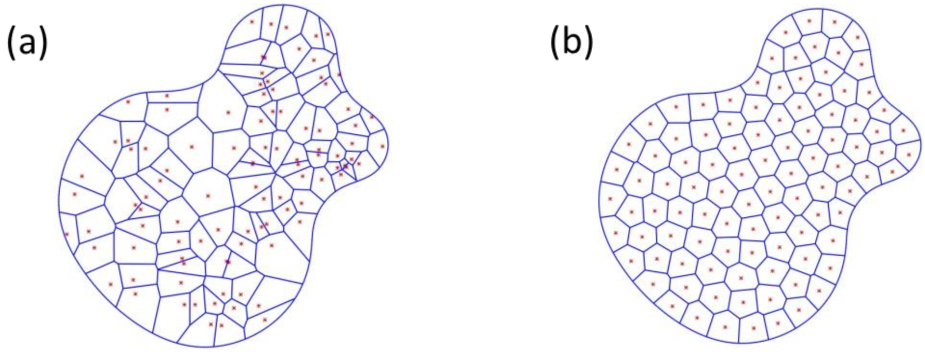

Fig. 1: Voronoi tessellations over the domain bounded by an arbitrary curve: (a) the general Voronoi tessellation with uniformly distributed generating points; (b) the centroidal Voronoi tessellation constructed from the general Voronoi tessellation.

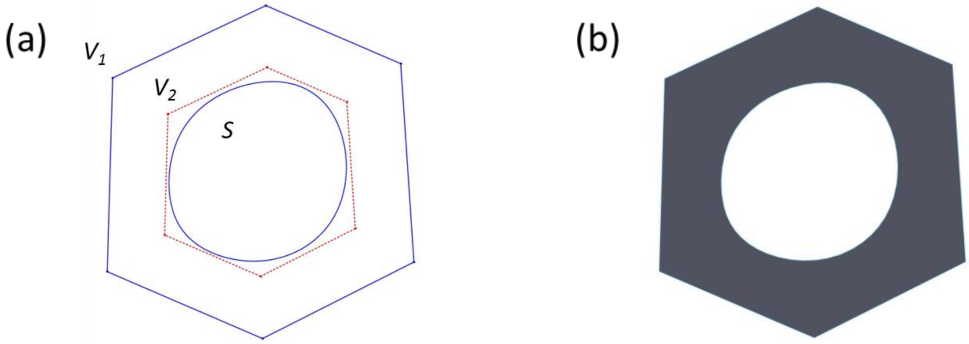

Fig. 2: The construction process of a pore within a single Voronoi cell: (a) construct a B-Spline curve with the vertices of the scaled Voronoi cell being the control points; (b) generate the pore by using Boolean operations.

Fig. 3. (a) shows the porous structure generated from the VT depicted in Fig. 1. (a), and Fig. 3. (b) presents the porous structure constructed from the CVT depicted in Fig. 1. (b). In this example, we set the scaling factor *t* as 0.5. Notice that, with a uniform density function, the pore size in the VT-based porous structure is uniformly distributed globally, but locally, there exists obvious differences of pore sizes. In the CVT-based porous structure, however, the pore size is uniformly distributed both globally and locally. As a result, our method performances better on local control of pore size and distribution. From the histograms in Fig. 3., we can also notice that the minimum angle (57.7°) within Voronoi cells of the CVT-based porous structure is larger than that of the VT-based porous structure (15.9°), while the maximum angle (139.2°) is smaller than that of the VT-based porous structure (177.5°). In light of that, our method can significantly impove the quality of the polygonal mesh embedded into the porous structure, and avoid overly large or small angles which will influence the stiffness matrix conditions and simulation results in VCFEMs [15].

In Fig. 3., the pore size and distribution is controlled by a uniform density functioin. Given a non-uniform density function, our method can easily construct irregular porous structure with non-uniform pore distribution, as shown in Fig. 4.. We can notice that similar observations in Fig. 3. Can also be found in Fig. 4.. The CVT-based porous structure is superior to the VT-based porous structure in terms of local pore size distribution and polygonal mesh quality (see the the histograms in Fig. 4.).

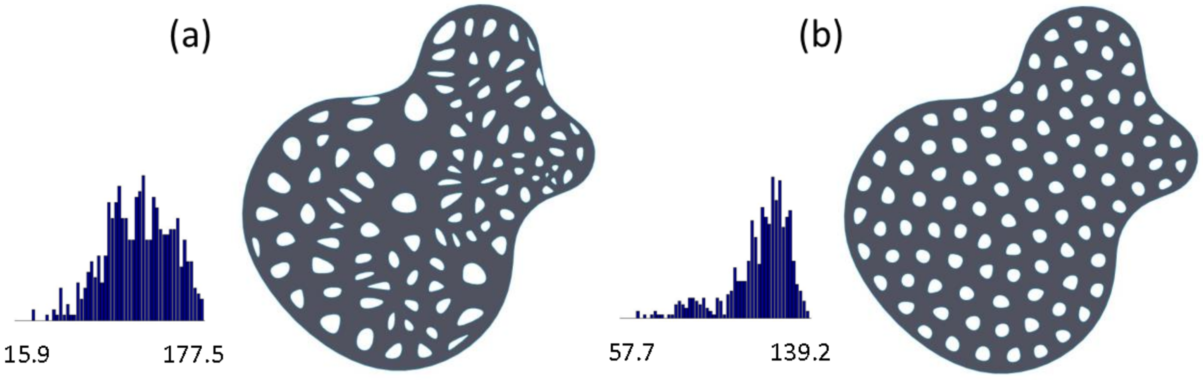

Fig. 3: The porous structures generated from the Voronoi tessellations depicted in Fig. 1 with the scaling factor *t* being 0.5: (a) the VT-based porous structure; (b) the CVT-based porous structure.

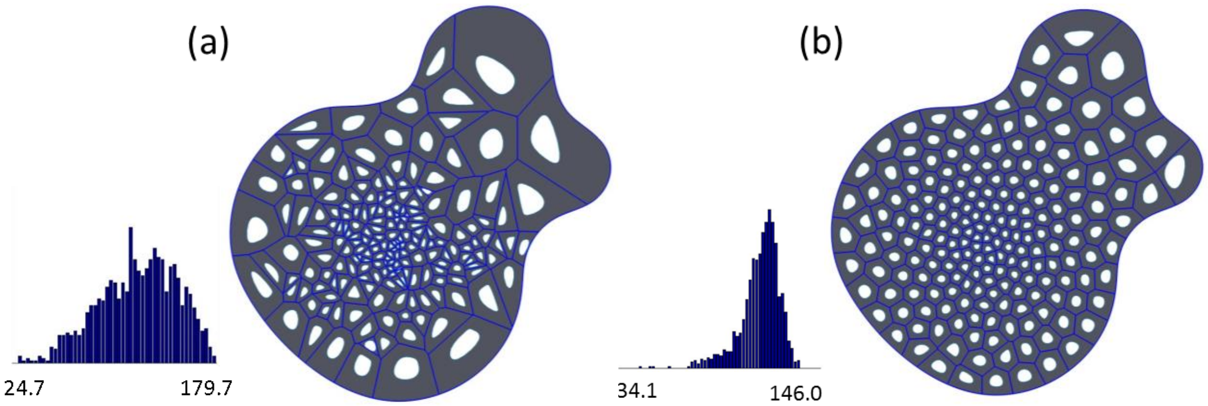

Fig. 4: Porous structures with respect to a non-uniform density function: (a) the VT-based porous structure; (b) the CVT-based porous structure.

Conclusion:

In this paper, a new method for porous structure modeling based on CVT and B-Spline is introduced. Benefiting from CVTs' superior properties, the proposed method generates porous structures with irregular pore shapes and diverse pore distributions according to a given density function. In addition, our method can significantly impove the quality of the polygonal mesh embedded into the porous structure, and avoid overly large or small angles which will influence the stiffness matrix conditions and simulation results in finite element analysis [15]. One weakness of this paper is that only 2D porous structure modeling is taken into account. In the future, we will extend this work to 3D porous structure modeling.

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