Title:
On the Interpolation of Non-Iso-Parametric Curves

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Introduction:
The particular choice of the title of the paper is motivated by the problem of modifying an initial B-spline surface to interpolate a non-constant-parameter B-spline curve [4]; a problem that is still defying an exact solution, even after a few decade of research. Moreover, the approximate solutions that have so far been developed for that problem are not sufficiently efficient for practical purposes. Furthermore, the particular wording of the above title is also decided in the spirit of the title of the above reference.

In this context, it may be argued that the main difficulty standing in the way of an exact and practical solution of the problem is the inability to incorporate the given curve as an integral part of the surface without destroying the regular B-spline nature of the surface, and therefore threatening the starting premise of the problem itself. This may also be taken as the source of the high degree of the curve-on-surface emanating from the original curve, and therefore resulting in the inability of exactly matching the original curve with the corresponding curve-on-surface [6], and consequently resulting in the inability of obtaining an exact solution of the problem.

In view of these difficulties, the present paper reduces the problem to that of constructing a Catmull-Clark subdivision surface [3] that can interpolate an initial regular cubic B-splince curve. The relationship between the original problem of [4] and the problem that is addressed by the present paper is still maintained by insisting that the control mesh corresponding to the initial given subdivision surface is regular, and that the control polygon corresponding to the initial surface is not necessarily equal in size nor parallel to any of the polygons constituting the control mesh of the initial surface.

In this sense, the work reported in this paper represents yet again another demonstration that adopting an alternative representation of a problem might render a defiant solution possible.

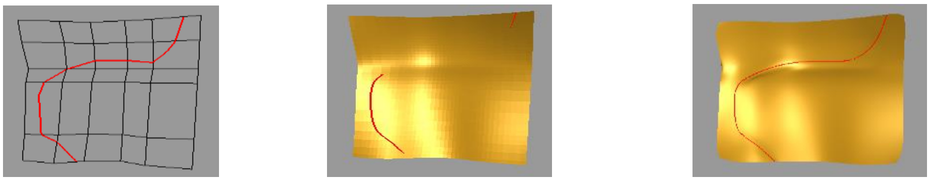

Fig. 1: The Interpolation Process: (a) Regular mesh and non-iso-parametric curve, (b) Curve not interpolated, and (c) modified Surface and Interpolation.

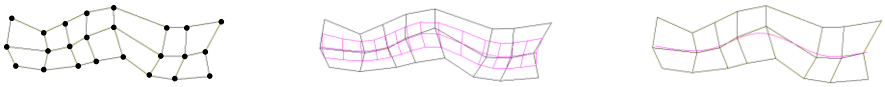

Fig. 2: Catmull-Clark polygonal complex and limit curve: (a) Initial Complex, (b) One Subdivision Step, and (c) Limit Curve.

The interpolation process makes heavy use of the notion of polygonal complexes [1], and Catmull-Clark polygonal complexes, in particular (see Fig. 2(a)).

The original motivation behind the conception of the notion of a polygonal complex [1] is that, under the corresponding subdivision scheme, it admits a B-spline limit curve of the same degree (see Fig. 2(c)). Thus, when a complex is embedded within a control mesh, its limit curve is automatically interpolated by the surface limit of subdivision of the polygonal mesh, without the need for any additional overheads (also see [7]). This should be contrasted with the classical ways of achieving interpolation (see [5], [9] and [10]).

Two further properties have also been developed in association with this notion:

- The nature of the limit curve of a Catmull-Clark polygonal complex (P): in fact, it turns out to be a uniform cubic B-spline curve (C) with a control polygon π(P) obtained as a result of applying a well-specified function π to the complex (P).
- Interpolating the curve whose control polygon is the mid-polygon of the complex (P)[8]: a complex τ(P), obtained by applying a well-specified transformation τ to (P), will have a limit curve corresponding to the mid-polygon of the (P) itself.

Main Idea:

Given an initial surface (S) (with a regular control mesh (M)) and an initial curve (C) (with a control polygon (P)) not necessarily oriented along any of the polygons constituting (M), the aim is to subject (M) to a sequence of transformations leading to a control mesh (M') that correspond to a subdivision surface (S') interpolating the initial curve (C).

This may be accomplished through the following sequence of steps:

- Integrate the polygon (P) within the mesh (M) by making sure that every next vertex of (P) sits on a next edge of (P). Thus, every next edge of (P) will split a next face of (M) into two faces (see fig. 1(a)).
- Remove all edges of (M) that may interfere with this scheme.
- Subject the resulting mesh (M1) to a single subdivision step. This gives rise to a polygonal complex (X1) embedded within (M1). At the same time, subdivide (P) separately one step leading to a polygon (P1).
- Replace the mid-polygon of (X1) within (M1) by (P1) point for point in the obvious way. This leads to a mesh (M2) and a complex (X2).
- Subject (X2) within (M2) to the transformation (τ), thus leading to a mesh (M3).
- This way, further subdivisions of (M3) would lead to the limit surface (S') interpolating the initial curve (C), as intended (see fig. 1(c)).

This algorithm necessarily affects the regularity of the initial control mesh, but without affecting its use for the underlying subdivision process

Further Benefits:

The above scheme may be extended through the use of X-configurations [2] to further interpolate partial (see fig. 3(a)) as well as intersecting curves (see fig. 3(b)). Until resolved, these topics are considered in [4] as obstacles in the way of progress of this line of research.

Furthermore, the many uses of individual subdivision steps in the above algorithms helped to shed some light on the origin of the concept of X-Configurations, which further validate their use for interpolating intersecting curves by subdivision surfaces.

Fig. 3: Further Benefits: (a) Interpolating partial curves, and (b) Interpolating intersecting curves.

Conclusions:

This paper presents an algorithm for the interpolation of B-spline curves that do not necessarily follow the directions of the base lines of the initial regular control mesh of the interpolating surface. The underlying algorithm relies on subdivision surfaces and polygonal complexes. This approach regards the solution as essentially a control mesh reconfiguration process. In contrast with previous research work that could be judged as relevant to the subject, the present approach results in precise (not approximate) interpolation, which does not require any changes to the subdivision coefficients except in well specified situations. Moreover, following this approach, efficiency does not present itself as an issue in the overall process.

Furthermore, in comparison with the work reported in [4] and in [6], the interpolated curve managed to retain its degree when placed on the surface. However, the area of the research, that has not received any attention in this paper (and therefore requires further work), is the quality of the resulting surface. This is obviously related to the appearance of extraordinary vertices on the control mesh during re-meshing and also to the particular decisions that should be taken when placing the extra points that are required during the process.

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