Title:
Filling the Empty Spaces of the Sierpinski Tetrahedron to Create a 3D Puzzle

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Keywords:
Sierpinski tetrahedron, puzzle

DOI: 10.14733/cadconfP.2014.4-6

Introduction:
Waclaw Sierpinski introduced in 1919 a fractal called Sierpinski triangle. This fractal was developed by Sierpinski from the interaction of an equilateral triangle, where the interaction n=0 is the area of the equilateral triangle of side L, the interaction n=1 is the midpoint of side L, i.e. $\frac{L}{2}$, the interaction n=2 is where each of the triangles has a length $\frac{L}{4}$, and so on. At each interaction the central triangles are extracted leaving empty spaces, and the filled triangles are connected by the vertices (Fig.1 (a), (b) and (c)). The Sierpinski tetrahedron (Fig.1 (d)) is the three-dimensional shape of the Sierpinski triangle and is used the same procedure for its construction, but the difference is that in the empty spaces in the Sierpinski tetrahedron we found equal irregular shapes of different sizes (Fig.1 (e)).

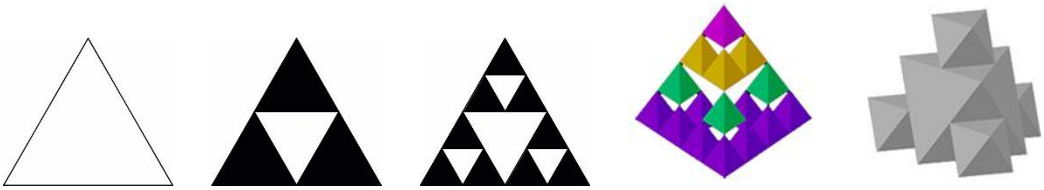

Fig. 1: Interactions of Sierpinski triangle: (a) n=0, (b) n=1 and (c) n=2; Sierpinski tetrahedron (d) Model and (e) Irregular shapes of the empty spaces.

This project stems from the idea of filling, with tetrahedral dimension $\frac{L}{4}$, the empty spaces of Sierpinski tetrahedron in its second stage, i.e. in the n=2 interaction to build a 3D puzzle using the following methodology.

- Development of the Sierpinski tetrahedron in its second stage.
- Analysis of unions and intersections of the tetrahedrons.
- Generation of the modules.
- Assembly of the 3D Puzzle.

Main Idea:
The purpose of this paper is to present the method used to fill, with tetrahedrons of dimension $\frac{L}{4}$, the empty spaces of the Sierpinski tetrahedron. And the aim of this research is to create a 3D puzzle using modules that bind and / or intersect each other.

As example of the 3D tetrahedral puzzles, wherein modules are used, we found the puzzles designed by James Allwright [6] (intersection of a truncated tetrahedra), and Wayne Daniel [7] (tetrahedrons cut). The difference between the 3D tetrahedral puzzles designed by Allwright and Daniel and the 3D tetrahedral puzzle presented in this research is that the tetrahedrons in each of the modules is based on the dimension $\frac{L}{4}$ of the Sierpinski tetrahedron, i.e. one quarter of the length of the edge, and in each of the modules took into account the unions between two tetrahedrons, the intersections between two or three tetrahedrons, and the empty spaces to insert a tetrahedron for assembling the 3D puzzle.

The 3D puzzle (Fig.2) is composed of 10 modules, and the modules (Fig. 3) are formed by 2,4, 5,7 and 8 tetrahedrons dimension $\frac{L}{4}$ respectively, as an example, it can see in Fig. 4 and Tab. 1, how to create a module using unions and intersections of four tetrahedrons to create an empty space so that a tetrahedron inserted into the open space and it can assembly the 3D puzzle.

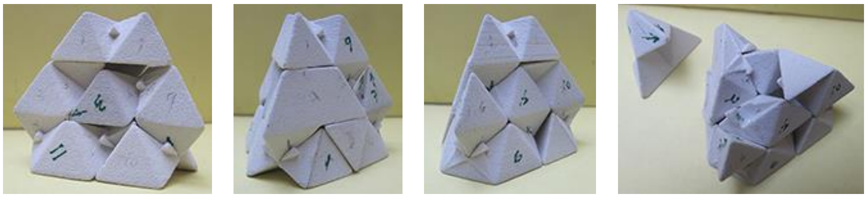

Fig. 2: 3D puzzle, (a, b, c) Exterior views and (d) Interior view.

Fig. 3: Modules.

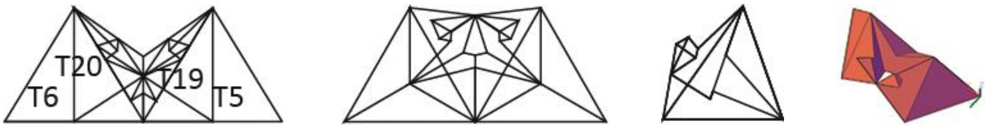

Fig. 4: Module four using four tetrahedrons: (a, b and c) Orthogonal projection, (d) Render.

Modules	Tetrahedrons	Faces which bind	Tetrahedron that intersect.	Intersecting modules	Edges that join
4	T6,T19,T20 and T5	T6-T20 and T5-T19	T20-T19 (I5)	Module 8 I5-T31	NONE

Tab. 1: Module using four tetrahedrons.

Conclusions:

This paper presents a system to create a 3D puzzle by filling the empty spaces of the Sierpinski tetrahedron. The puzzle is made up of a number of modules and the modules are a series of tetrahedrons of dimension $\frac{L}{4}$ forming different shapes.

The results show that it is not possible to create a 3D puzzle completely filled because the tetrahedron is a solid formed by four equilateral triangles that meet at a vertex, and always, no matter the position of the tetrahedron in space maintains its inclination angle, and due to this inclination was achieved that the 3D puzzle can be assembled

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