Title:
A Computational Framework for the Boundary Representation of Solid Sweeps

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Introduction:
This paper is about the theory and implementation of the solid sweep as a primitive solid modeling operation. A special case of this, viz., blends is already an important operation and prospective uses for the sweep are in NC-machining verification [4], collision detection, assembly planning [1] and in packaging [7].

The solid sweep is the envelope surface $\mathcal{E}$of the swept volume generated by a given solid M moving along a one-parameter family h of rigid motions in $\mathbb{R}^3$. We use the industry standard brep format to input the solid M and to output the envelope $\mathcal{E}$. The brep of course has the topological data of vertices, edges and co-edges, loops bounding the faces and orientation of these, and the underlying geometric data of the surfaces and curves. As we show, the brep of $\mathcal{E}$, while intimately connected to that of M, has several intricate issues of orientation and parametrization.

Much of the mathematics of self-intersection, of passing body-check and of overall geometry have been described in the companion paper [5]. This paper uncovers the topological aspects of the solid sweep and its construction as a solid model. Here, we restrict ourselves to the simple generic case, i.e., smooth M and, therefore smooth $\mathcal{E}$ which is free from self-intersections, to illustrate our approach and its implementation. The general case is also implemented and a few sample sweeps appear in Fig. 1.

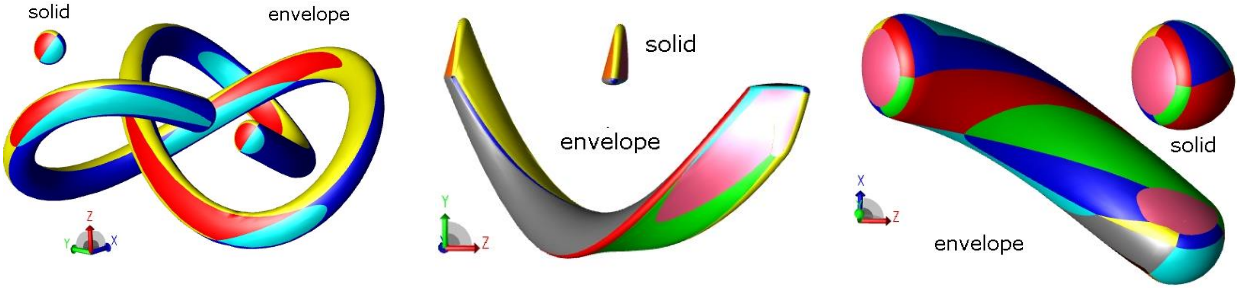

Fig. 1: Three examples of solid sweeps.

Background:
The solid sweep has been extensively studied [1,2,4,6,8]. Some of the prominent approaches are based on sweep envelope differential equation [4], Jacobian rank deficiency condition [2] and a point membership test using the inverse trajectory [6]. For a more comprehensive survey of the previous work, we refer the reader to [1]. Much of the work has focused on the mathematics of the surface. The exact topological structure has not been investigated in any significant detail.

Main contribution:
Our main contributions are (i) a clear topological description of the sweep, and (ii) an architectural framework for its construction. This, coupled with [5], which constructs the geometry/parametrizations of the surfaces, was used to build a pilot implementation of the solid sweep using the popular ACIS solid modeling kernel [3]. We give several illustrative examples produced by our implementation to demonstrate the effectiveness of our algorithm. To the best of our knowledge, this is the first attempt to explicate the complete brep structure of $\mathcal{E}$.

Central Idea:
The central idea is to resolve the face adjacency and orientation linkage between the solid boundary ∂M and the envelope $\mathcal{E}$ through the natural correspondence $\pi : \mathcal{E} \to \partial \mathcal{M}$ which associates to a point x on $\mathcal{E}$ the point $\pi(x) = y$ on ∂M whose 'translate' generated x. The map π is illustrated in Fig. 1 and Fig. 2 using colour-coding, i.e., the face F of ∂M and the faces of $\mathcal{E}$generated by F are shown in same colour.

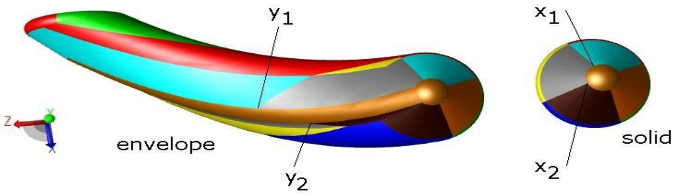

Fig. 2: The envelope of a saucer being swept along a helical trajectory with compounded rotation.

We show that the adjacency relations between vertices, (co-)edges and faces of $\mathcal{E}$ mimic those of ∂M. For instance if F' and G' are faces of $\mathcal{E}$ with corresponding faces F and G of ∂M and further, if F' and G' are adjacent via an edge e', then F and G are adjacent via an edge e which corresponds to e' (see Fig. 2). Similarly, if two co-edges of $\mathcal{E}$ intersect in a vertex then the corresponding co-edges of ∂M necessarily intersect in the corresponding vertex. This allows for a guided computation of the 'unoriented' topological 2-skeleton (vertices, edges, faces and their adjacencies) of $\mathcal{E}$, locally lifting the 2-skeleton of ∂M.

It turns out that, in general, the map π is fairly intricate since it can be both orientation preserving and reversing at different points. For instance, in Fig. 2, vertices labeled y_1 and y_2 in $\mathcal{E}$ correspond to x_1 and x_2 in ∂M. The map π is orientation reversing at y_1 while it is orientation preserving at y_2, as can be seen by the color of the adjacent faces. Our main theorem provides a complete characterization of the regions on $\mathcal{E}$ where π is orientation preserving and reversing respectively, thereby, enabling us to lift orientations of faces and co-edges of ∂M to those of corresponding faces and co-edges of $\mathcal{E}$ respectively. This completes the construction of the oriented topological 2-skeleton of $\mathcal{E}$.

For a face F of ∂M, let D be the subset of parameter space of surface S underlying face F so that $S(D) = F$. Let the closed time interval I be the domain of trajectory h. We refer to the set $D \times I$ as the prism. The envelope condition (related to the Jacobian rank deficiency condition [2]) gives rise to a 2-dimensional submanifold of the prism, which we refer to as the funnel (see [5]). The funnel serves as a parametrization space for the faces of $\mathcal{E}$ corresponding to F. Further, the intersection of the funnel with the boundary of the prism serves as a parametrization space for the co-edges of $\mathcal{E}$ which bound faces corresponding to F.

Our algorithm is based on the following architectural framework. Before any geometric entity of $\mathcal{E}$ is computed, its boundary is computed and oriented. First, we compute the 0-skeleton, i.e., the vertices of $\mathcal{E}$. This is followed by computation of 1-skeleton, i.e. oriented co-edges and loops which will bound faces of $\mathcal{E}$. Finally, the faces are oriented and parametrized as described above to produce the complete Brep of $\mathcal{E}$.

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Conclusion:

We explicate the complete Brep of the solid sweep as a primitive solid modeling operation. Further, we provide a novel algorithmic framework for its computation. We give several illustrative examples generated by a pilot implementation of our algorithm to demonstrate the efficiency of our method.

References:

[1] Abdel-Malek, K.; Blackmore, D.; Joy, K.: Swept Volumes: Foundations, Perspectives and Applications, International Journal of Shape Modeling, 12(1), 2006, 87-127. http://dx.doi.org/10.1142/S0218654306000858

[2] Abdel-Malek, K.; Yeh, H.J.: Geometric representation of the swept volume using Jacobian rank-deficiency conditions, Computer-Aided Design, 29(6), 1997, 457-468. http://dx.doi.org/10.1016/S0010-4485(96)00097-8

[3] ACIS 3D Modeler, SPATIAL, http://www.spatial.com/products/3d_acis_modeling

[4] Blackmore, D.; Leu, M.C.; Wang, L.: Sweep-envelope differential equation algorithm and its application to NC machining verification, Computer-Aided Design, 29(9), 1997, 629-637. http://dx.doi.org/10.1016/S0010-4485(96)00101-7

[5] Bharat, A.; Jinesh, M.; Milin, S.: Local and Global Analysis of Parametric Solid Sweeps, Cornell University Library arXiv. 2013. http://arxiv.org/abs/1305.7351

[6] Huseyin, E.; Ilies, H. T.: Classifying points for sweeping solids, Computer-Aided Design, 40(9), 2008, 987-998. http://dx.doi.org/10.1016/j.cad.2008.07.005

[7] Kinsley Inc. Timing screw for grouping and turning. https://www.youtube.com/watch?v=LooYoMM5DEo

[8] Peternell, M.; Pottmann, H.; Steiner, T.; Zhao H.: Swept volumes, Computer-Aided Design and Applications, 2(5), 2005, 599-608. http://dx.doi.org/10.1080/16864360.2005.10738324