Title:
T-Spline Polygonal Complexes

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Introduction:
The earliest notion of polygonal (strip) complexes was proposed by Nasri [9] as a means of interpolating uniform quadratic B-spline curves by Doo-Sabin subdivision surfaces [4]. Later on, Nasri [8] redeployed this notion to also support the interpolation of uniform cubic B-spline curves by Catmull-Clark subdivision surfaces [3].

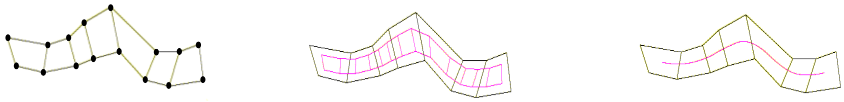

Fig. 1: Doo-Sabin Polygonal Strip Complex and Limit Curve: (a) Initial Complex, (b) One Subdivision Step, and (c) Limit Curve.

The initial motivation behind a polygonal complex [1] is that, under the corresponding subdivision scheme, it admits a B-spline limit curve of the same degree (see Figures 1 and 2). Thus, when a complex is embedded within a polygonal mesh, its limit curve is automatically interpolated by the surface limit of subdivision of the polygonal mesh, without the need for any additional overheads.

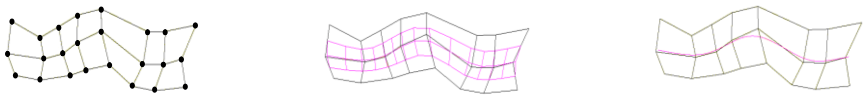

Fig. 2: Catmull-Clark Polygonal Complex and Limit Curve: (a) Initial Complex, (b) One Subdivision Step, and (c) Limit Curve.

More interestingly, as suggested in the previous paragraph, nothing prevents the extension of this notion to any other subdivision scheme [6].

This notion was employed later, under well-specified constraints [3], for the interpolation of any arbitrarily intersecting network of curves by a subdivision surface (see Figure 3). This works for Doo-Sabin [7] and Catmull-Clark [2] as well as for Loop [5] subdivision schemes.

Polygonal complexes can also serve as local neighborhoods for holding information useful for exercising further control over the limit surface; e.g., normal direction and local curvature constraints.

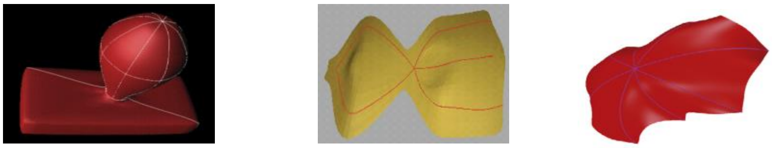

Fig. 3: Interpolating an Intersecting Network of Curves by a Subdivision Surface: (a) Doo-Sabin, (b) Catmull-Clark, and (c) Loop Subdivision Surface.

Main Idea:

The main goal of the paper is to be able to interpolate B-spline curves by T-spline surfaces directly through the application of a single mathematical formula in much the same way that has been successfully done in the context of Doo-Sabin, and Catmull-Clark Subdivision Surfaces and later on in the context of Loop subdivision Surfaces.

To this end, the paper proceeds from the basic definition of B-spline curves and surfaces [11] and managed to derive a formulation of Polygonal Complexes for B-spline surfaces.

The next step was to generalize that formulation further so as to cope with the needs of the NURBS domain [10]. The ultimate step was to generalize that even further so as to be able to cope with the needs of the T-spline domain.

This last derivation could not have been done without an enhanced implementation of the T-spline local refinement algorithm from that which was originally published by Sederberg et al [12].

The paper provides an explicit formulation of T-spline polygonal complexes and provides illustrations to show its success in interpolating of B-spline curves by T-spline surfaces.

Conclusions:

The usefulness of polygonal complexes, beside many other things, resides in their ability to achieve interpolation at very little computational cost, with relatively simple operations, and without having to call in heavy mathematical weaponry. Other important uses of these complexes have been discussed in the introduction and also in main body of the paper.

Considering the generality of the approach, this work should be expandable to cover more liberal domains, but perhaps without being able to make use of matrices. In fact, the polygonal complex may not always fit into the confinement of the rectangular space of a matrix, because the approach will have to cater for the domain of influence of every point of the central polygon of interest, which may follow any arbitrary pattern; i.e., it may not necessarily be rectangular every time.

Further research direction could go into the interpolation of partial and of non-iso-parametric curves, particularly in the context of T-splines.

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