Title:
Investigations on Different Geometric Elements for Quadtree-based Scanning of 2-D Spaces

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Motivation:
Quadtrees are hierarchical data structures which are based on the principle of recursive sub-division of space. Quadtrees date back to 1970s [3], and since then they have been used heavily for representation of spatial data. One of the prime motivations of using hierarchical quadtrees over other techniques is to save memory space, as well as to improve the computational efficiency of computational procedures. Different geometric elements have been used to decompose space in terms of quadtrees, such as rectangles, triangles, and their higher-dimensional counterparts [1]. The computational complexity in triangular sub-division is the same as in rectangles, but the triangular subdivision may perform better for capturing the boundaries of certain regions, since scanning in triangles takes place for every 60° compared to the 90° in the case of squares or rectangles.

This paper proposes a novel decomposition technique for quadtrees in the form of circular segments for representation of 2D data. Rectangular, triangular, and the newly suggested circular subdivision techniques were tested to detect the traces of implicitly defined planar curves. This has immense utilities in the domain of mechanism design and analysis [2], among others. Comparisons between results obtained by the three different geometric elements in terms of computational time, accuracy of the result obtained, and memory consumed. It is found, in general, that the RMS error of the obtained results reduced faster in the case of the circular segments, than in the other two cases.

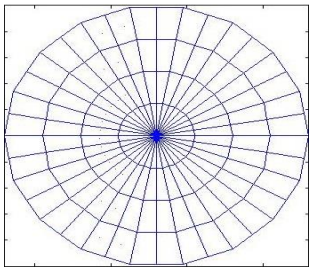
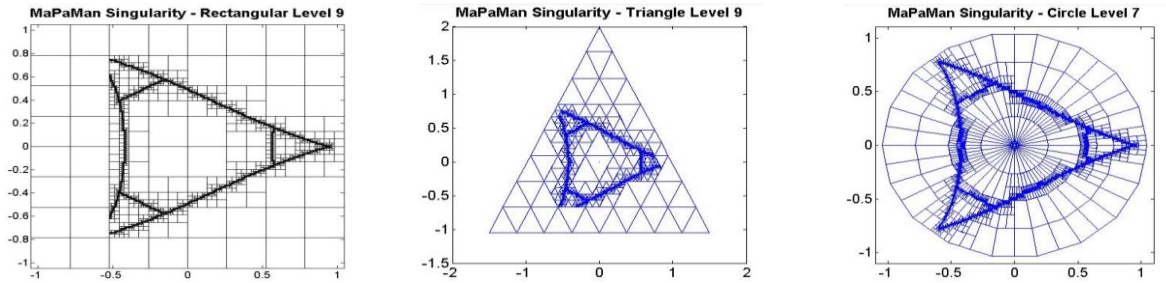


Fig. 1: Grid for Circular element decomposition.

Methodology:
A quadtree decomposition, as the name suggests, uses a strategy of dividing a “parent” element into four equal sub-elements at each stage or iteration, till the required resolution/accuracy is achieved. An extensive overview of the general scheme is available in [4]. The rectangular and triangular elements

used in this paper follow the same methodology. A depth-first traversal model has been adopted in this work. In the case of circular elements, a rectangular grid is considered in the polar space, and subsequently mapped to the Cartesian plane, as shown in Fig. 1.

Results:
The proposed methodology was first tested on a number of simple planar curves initially, and then on used for detecting a number of complicated curves known as the "singularity curves", S1, S2, of a parallel manipulator, namely, the MaPaMan, reported in [2]. The three geometric elements namely rectangle, triangle and circle were used as the quadtree primitives, and starting from the same region of interest in the data space the quadtree were generated to at least seven successive subdivision. The results are presented in Fig. 2(a), 2(b), 2(c) respectively.



(a) (b) (c)

Fig. 2: Plots of S1, S2 functions of MaPaMan.

Discussion:
To compare the performance of the quadtree with the three geometric primitives, namely rectangle, triangle and circle, measures based on error in representation and computational efficiency were considered. The error measure chosen is the RMS deviation of the grey cell's (the boundary cells that get divided at each subsequent stage of subdivision) centroids from the actual region boundary. Computational efficiency is analyzed using the CPU time as well as the number of function evaluations. Fig. 3-5 depicts these performance measures for the three different geometric elements for the case study described in Fig.2. The decrease in RMS error measure with subdivision is similar for quadtree with rectangle and triangle primitives whereas the decrease in RMS is better for subdivision with circular elements (as shown in Fig. 3). From Fig. 4 it is inferred that the CPU time versus function evaluation is similar for rectangular and circular elements whereas it is different for the triangular elements. This is attributed to similar topology of the rectangular and circular geometric elements. Error versus function evaluation analysis (Fig. 5) shows that rectangular and circular elements fair better than triangular elements.

Similar case studies have been done on several other functions such as singularity function of a 3RRR planar manipulator, analytical functions like rotated and translated rectangular hyperbola and ellipses. Similar comparative results have been obtained to support the above inferences. The study based on the case studies performed, summarizes that the proposed quadtree generation using circular geometric elements results in better boundary detection for similar computational effort as that of quadtree with rectangular element and always better than that of quadtree with triangular elements.

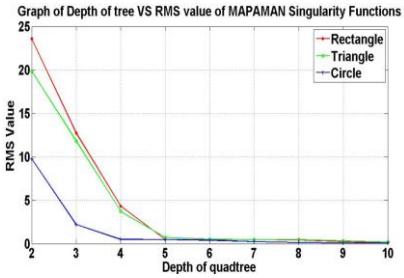


Fig. 3: Plot of Depth of tree VS RMS Value

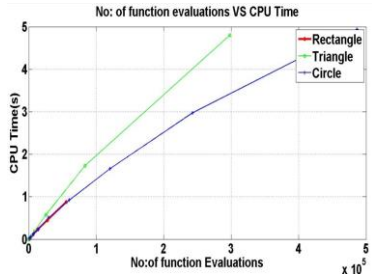


Fig. 4: Plot of No: of func eval VS CPU Time

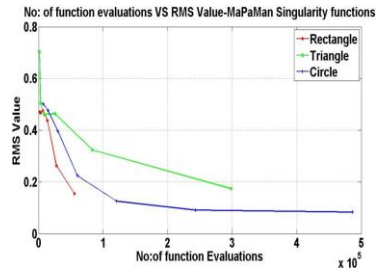


Fig. 5: Plot of No: of func eval VS RMS Value

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