Title:

A Heuristic Offsetting Scheme on Catmull-Clark Subdivision Surfaces

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Introduction:

In rapid prototyping, a hollowed prototype is preferred and significantly reduces the building time and material consumption in contrast to a solid model. Most rapid prototyping obtains solid thin shell by gradually adding or solidifying materials layer by layer. However this is a non-trivial problem to offset a solid involving finding all self-intersections and filling gaps after raw offsetting. While Catmull-Clark subdivision (CCS) [1] surfaces are widely used in solid modeling, the hollow solid/thin shell problems are not well addressed yet. In this paper, we explore the earlier methods of obtaining thin shell CCS solid and present a new thin CCS thin Solid approach, with this new scheme, one can efficiently avoiding the filling gaps and self-intersections with earlier schemes.

The new scheme is heuristic, but inner surface is parametric, such that computation of inner surface is simplified. And with Bezier crust applied, inner surface maintains the mesh structure and continuity of outer surface. The obtained thin shell solid is C^2 everywhere except at extraordinary points, where it is C^1 continuous.

Main idea:

Catmull-Clark subdivision surfaces have been widely used in computer graphics and animation. In contrast to traditional spline schemes, the CCS scheme can handle arbitrary topology and easy to design and implement. In 3D modeling, building a hollowed prototype instead of a solid model is required to reduce the building time and material consumption. When we use CCS to generate a hollowed object, intuitive way is to construct CCS meshes for outer and inner surfaces. However, it is not effective and many issues are raised during construction of inner surfaces, e.g. surface collision, self-intersections. It is not an easy task to design a CCS control mesh to generate a thin-shell hollowed 3D object.

In this paper, we present a C^2 offsetting scheme on CCS surfaces. With this new scheme, one can generate hollow 3D solids efficiently with one layer of CCS control mesh and maintain the curvature continuity of CCS scheme. Earlier hollowing methods of octree and voxel model, constructive solid geometry offsetting and curve offsetting [4] [5] are not applicable, since inner surfaces generated with these offsetting methods are not C^2. Due to the parametric properties of CCS, in our new scheme, we use a new surface offsetting approach, which offset the limit surface directly by adding a thin layer of bi-quintic Bezier surface.

In Wang and Cheng 2013 [3], a Bezier crust scheme is applied to CCS limit surface to obtain a parametric interpolating surface, the bi-quintic Bezier crusts added will maintain the curvature continuity of underlying CCS parametric surfaces. This is consistent with research of Kahmann 1983 [2]. The scheme of Bezier crust works on difference vectors between control points and their corresponding data points. Here we show how this concept can be applied to offset the CCS surface.

Given a CCS control mesh D, we have a set of control points V_i, i=1,..n. With CCS, we obtain limit surface with corresponding data points at four corners d_i, i=1,..,n and their unit normal n_i. If we set the thickness of desired thin-shell as c, then we can define a set of difference vectors of cn_i, (d_i-cn_i) will be the desired corner data points on the offsetting surface. When we apply Bezier crust on these difference vectors at four corners of a CCS face, we can obtain a parametric offsetting surface having uniform distance of cn_i at all corners of each CCS face while keeping CCS continuity after offsetting. The offsetting parametric surface O(u,v) obtained can be expressed as follows:

$$O(u,v) = S(u,v) - \nabla p(u,v) \qquad (1)$$

Where $\nabla p(u,v)$ is the Bezier crust on difference vectors of cn_i, with expression of

$$\nabla p(u,v) = \sum_{i=0}^{5} \sum_{j=0}^{5} \mathrm{b}_{\mathrm{i},5}(\mathrm{u})\, \mathrm{b}_{\mathrm{j},5}(\mathrm{v}) \nabla P_{i,j} \qquad (2)$$

With $\nabla P_{i,j}$ takes value of ∇P_0, $\nabla P_1, \nabla P_2$ *and* ∇P_3. $\nabla P_{i,j} = \nabla P_0$ if $\in [0,2]$ & $j \in [0,2]$; $\nabla P_{i,j} = \nabla P_1$ if $i \in [0,2]$ & $j \in [3,5]$; $\nabla P_{i,j} = \nabla P_2$ if $i \in [3,5]$ & $j \in [0,2]$; $\nabla P_{i,j} = \nabla P_3$ if $i \in [3,5]$ & $j \in [3,5]$. ∇P_0, $\nabla P_1, \nabla P_2$ *and* ∇P_3 are the difference vectors of cn_k at four corners of a CCS face (Fig. 1).

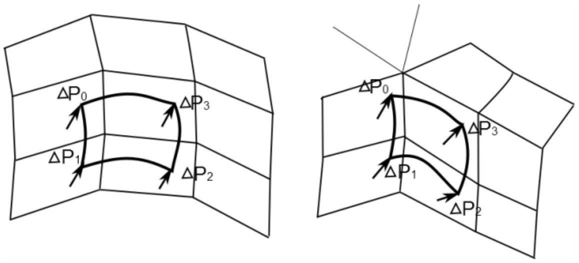

Fig. 1: difference vectors of $\boldsymbol{\nabla P_0}$, $\boldsymbol{\nabla P_1}, \boldsymbol{\nabla P_2}$ ***and*** $\boldsymbol{\nabla P_3}$ on a regular face (left) and an extraordinary face (right).

Implementation result in Fig. 2 shows that a smooth thin offsetting surface can be generated by applying our new scheme. The offsetting surface keeps a quasi-uniform thickness with CCS surfaces, which formed a nice hollow solid appropriate for common CAD usage. The scheme is based on the assumption that all corner CCS data points have non-zero unit normal, we will also include discussion of scenario when unit normal does not exist (control mesh collapses).

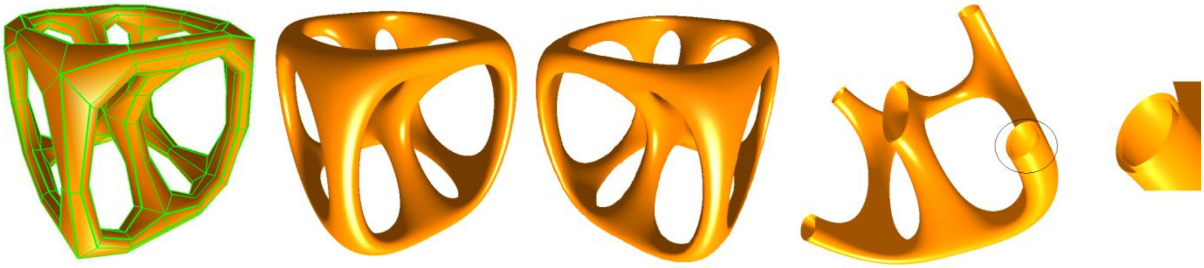

Fig. 2: a) CCS mesh, b) outer surface, c) offsetting surface, d) cross-section view, e) enlarged detail from cross-section.

Conclusion:

In this paper, we present a new heuristic surface offsetting scheme on CCS surfaces. With new scheme, one can generate a smooth offsetting surface with the same continuity as its CCS outer surface, such that 3D hollowing objects can be modeled from single layer of CCS mesh efficiently.

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