Title:
Tolerance Mathematical Modeling and Analysis Method Based On Control Points of Geometric Elements

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Introduction:
In this paper, the tolerance mathematical modeling and analysis method based on control points of geometric elements are proposed, where the geometric tolerance is indicated according to the position variation of control points of geometric elements.

This method can be applied in actual tolerance analysis of parts. The calculation results show that the mathematical modeling and analysis methods are in accordance with all relevant tolerance principles and regulations, which have a vital significance in the studying of whole tolerance field [1]. It can also analyze various corresponding tolerance, which can be easily put in combination with CAD entity modeling. It is convenient for tolerance analysis and conversion that can be applied to computer aided tolerance designing and has extensive application in the field of tolerance analysis [2].

Main idea:
The tolerance analysis method is used to control the degree of freedom, which deviating the ideal state target geometry elements on the size, location and shape. Compared with the rigid elements, the number of geometric elements DOFs have less than six due to the uncertainty: the factors including three translational DOFs T_1, T_2 and T_3. The line elements including two translational DOFs T_1, T_2 and two rotational DOFs R_1, R_2. The planes elements including along the normal of the translational DOF T and two rotational DOFs R_1, R_2 [3][4], which are showed in figure 1:

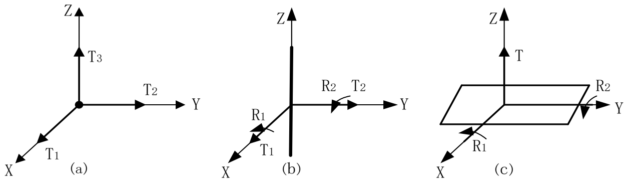

Fig. 1: The DOFs and position parameters of geometrical elements.

According to the error definition, the tolerance type of point elements only has position error. When the point hangs from P_0 to P_i, the tolerance can be expressed as:

$$L_{SL} \le \sigma = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2} \le L_{SU} \tag{1}$$

The L_{SU} and L_{SL} are upper and lower deviation respectively and the constraint conditions are:

$$\begin{cases} L_{\mathrm{SLX}} \le x_i - x_0 \le L_{\mathrm{SUX}} \\ L_{\mathrm{SLY}} \le y_i - y_0 \le L_{\mathrm{SUY}} \\ L_{\mathrm{SLZ}} \le z_i - z_0 \le L_{\mathrm{SUZ}} \\ \max\ (x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2 \end{cases} \tag{2}$$

The point tolerance domain has a variety of forms such as straight line, round, rectangular and cylindrical. In the rectangular coordinate system, the position point itself simulates the business field through three parameters. The linear elements can consider its two endpoints P_1P_2 as the tolerance control points. In the rectangular coordinate system, the variation of coordinate parameter is the tolerance value and the tolerance size can be marked as following:

$$\mathrm{D}_{\mathrm{SL}} \le \sigma = \sqrt{(\Delta \mathrm{x})^2 + (\Delta y)^2 + (\Delta z)^2} \le \mathrm{D}_{\mathrm{SU}} \tag{3}$$

Where:

$$\begin{cases} \Delta \mathrm{x} = \dfrac{(x_1' + x_2') - (x_1 + x_2)}{2} \\ \Delta \mathrm{y} = \dfrac{(y_1' + y_2') - (y_1 + y_2)}{2} \\ \Delta \mathrm{z} = \dfrac{(z_1' + z_2') - (z_1 + z_2)}{2} \end{cases} \tag{4}$$

The direction error of the line can use direction vector angle as following:

$$\theta_{\mathrm{SL}} \le \alpha = \arccos \frac{\left| aa' + \mathrm{bb}' + \mathrm{cc}' \right|}{\sqrt{a^2 + \mathrm{b}^2 + c^2}\sqrt{a'^2 + \mathrm{b}'^2 + c'^2}} \le \theta_{\mathrm{SU}} \tag{5}$$

The S (a, b, c) and S'(a, b, c) are the vectors of direction before and after change respectively. Graphic elements control parameters are the variation along the normal direction by three control points in the arbitrary plane.

<u>Mathematics:</u>

The application of control points of geometric elements in tolerance mathematic modeling can also be extended into the PSO algorithm. For example, the discrete measured points elements are be used to calculate the shape error. According to the minimum zone method, assessing shape error is a optimization problem in essence based on the PSO algorithm. The definition of shape error can be described in the extremum method.

$$\begin{aligned} &\min[\max F(U,V) - \min F(U,V)] \\ &\min F(U,V) \\ &\max F(U,V) \end{aligned} \tag{6}$$

Where F (U, V) is the objective function of shape error calculation and U is shape function of ideal geometry elements, X is the actual position function of geometric elements as follows:

$$\mathrm{V} = [f(x), f(y), f(z)] \tag{7}$$

The shape error is the change of single actual measured elements relative to the ideal ones. According to the definition for form error, its evaluation can only be carried out under the minimum condition. The tolerance zone shape can be judged according to the specific design requirements, calculating the tolerance objective function expression of each project. Its shape is the same as the ideal elements to

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determine the minimum position and direction. The tolerance area corresponding to the size of the minimum area is the shape error.

Conclusions:

The control point of geometrical elements can be got through the computer-aided tolerance analysis and the corresponding tolerance type and size can also be obtained. At the same time, the geometrical elements of control points can be combined with the PSO algorithm to establish the tolerance mathematical modeling. Compared to other tolerance analysis methods, this method have more advantages in the data storage structure, calculation and precision.

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