Title:
Interpolating Curved Grids using Quasi-developable Bi-cubic Gregory Patches

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Keywords:
Developable surface, Gregory surface, surface interpolation, curve network

DOI: 10.14733/cadconfP.2014.149-151

Introduction:

A surface is developable if it can be flattened onto a plane without distortion, i.e., without any stretching or compression. Developable surfaces are widely used in the engineering fields such as ship-building [9], garment manufacture [6, 11, 12] etc. Much work has been done on how to represent a developable surface using polynomial patches. One approach to achieve this is to use developable or quasi-developable polynomial patches to interpolate a set of boundary curves [1, 2, 4, 5-7] with G^0, G^1 and/or G^2 continuity. Another approach is to use piecewise linear approximations: the free-from surface is tessellated, typically using triangles, and an optimization model is constructed to minimize some form of distortion of the mesh in a 2-parameter space [5, 12]. Since a developable surface can be regarded as a one parameter family of straight lines, it can be mapped to one space curve in the dual space. Some researchers [3, 10] convert the modeling of developable surfaces by an equivalent modeling of a curve in dual space. This conversion can guarantee that the final surface is developable; unfortunately, the modeling process is not geometrically intuitive and therefore not popular among designers.

The aforementioned methods are not directly designed to interpolate a loop of four arbitrary boundary curves. Our objective is to find a quasi-developable surface to interpolate four given boundary curves, while ensuring G^1 continuity along the shared boundaries of adjacent patches. A loop with arbitrary number of boundary curves can be divided into a collection of quadrilateral patches with free form space lines. We can then construct quasi-developable surface for each of these quadrilateral patches.

Main idea:

The bi-cubic Gregory patch is adopted to model the interpolation surface. As the first and second cross-derivatives along the boundary of the Gregory patch are equal to the corresponding derivatives of a Bezier patch with the same boundary, we can map the task of computing the internal control points to that of computing the Bezier form control points for each boundary.
The G^1 continuity of two adjacent Gregory patches is only related with the first order derivatives along the common boundary. Therefore, we can make the two adjacent patches satisfy the G^1 continuity condition by constraining the location of the inner control points that are related to these first order cross derivatives. Note that if these control points are completely determined, the Gaussian curvature values at each of the corners of the patch are also fixed. The developability of the corner thus cannot be optimized in the subsequent optimization process. So we should optimize the developability of the interpolating surface around the corners first, before enforcing the G^1 continuity condition. By minimizing the magnitude of the Gaussian curvature at the corner points, we implicitly force the neighborhoods of the corners to be developable. Finally, we can calculate the first order cross

derivatives related control points by using the relationship of the first order derivatives of the Bezier patch's boundary between the control points. In order to fully determine all inner control points, we totally have eight free parameters which allow us to optimize the surface for developability.

Farin's method [8] is used to calculate the initial positions of inner control points, which in turn estimates the initial values for these eight free parameters. The metric to estimate the developability of the surface is a commonly adopted measure, namely, the integral Gaussian curvature over the interpolation patch. As the optimization function of the Gregory patch is a second order differential form polynomial, it is not amenable to an analytical solution. We choose the BFGS-quasi-Newton algorithm to numerically solve the optimization formulation.

Fig. 1 shows a car hood formed using sheet-metal. The shape is preferred to be as developable as possible so that the internal stresses are minimized. In this example, ribbon curves are constructed first making 24 curved grids. Zebra-pattern rendering and Gaussian map are adopted to visually verify the G^1 continuity and developability of the resulting surface. The results by benchmark and our algorithm are shown respectively in Fig. 1(b) and Fig. 1(c). From Fig.1, we can see clearly the effectiveness of our method in producing a more developable surface. The final composite surface is G^1 continuous.

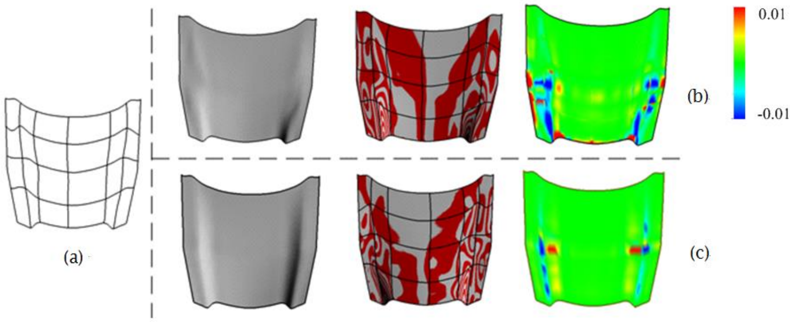

Fig. 1: (a) A curve network with 24 curved grids, each of which is bounded by four planar curves of degree 3; (b) the result by [8]; (c) the result by the proposed algorithm. The colored images on the right side show the Gaussian curvature map.

<u>Conclusions:</u>

In this paper, a novel algorithm is proposed to construct a degree-3 Gregory patch interpolating a curved grid consisting of four Bezier curves of degree 3. In this algorithm, the first cross derivatives along boundary curves are optimally specified by imposing a developability constraint and all inner unknown control points are evaluated by ensuring G1 continuity. The developability constraint used in our implementation minimizes the integral of the magnitude of the Gaussian curvature over the whole patch; other similar criteria can be easily adopted into our approach. Test examples showed that the proposed algorithm can be applied to many industries in which developability is desirable.

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