Title:
An Extension Algorithm for Disk B-Spline Curve with G^2 Continuity

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Introduction:
Disk B-Spline curve has its distinct advantages in representing a 2D region. In 2004, Wu and Seah et.al first proposed the disk B-Spline curve [3] through extending disk Bezier curve, which is defined in B-Spline form and describes not only a 2D region, but also the center curve (skeleton) of the region explicitly. Furthermore, DBSC is used for representing artistic brushstrokes and applications in 2D animation.

Curve extension is a general problem in curve design. A given curve usually needs to be extended in order to meet some geometric shape conditions or engineering requirements. Ting et al. proposed a DBSC extension algorithm based on curve unclamping [4]. With their method, the unclamped knot vector of the extended DBSC is computed according to the accumulated chord length method. The original DBSC and the extending segment satisfy C^2-continuity at the joint disk. However, the extended DBSC is exclusive and in some cases is not a desired result. Geometric continuity plays an important role in depicting curves' fairness and can supply additional degrees of freedom for adjustment. In this paper, we propose a G^2-continuity extension algorithm for DBSC. The shape of the extended DBSC is determined by minimizing its strain energy. The unclamped knot vector is determined by the optimal degree of freedom acquired. It is important for DBSC modeling in 2D animation and many other areas.

Main Idea:
Definition of disk B-Spline curve

A p-degree disk B-Spline curve is defined as $\langle D\rangle(t)=\sum_{i=0}^{n} N_{i,p}(t)\langle P_i;r_i\rangle$, where P_i is control point and r_i is control radius. $\langle P_i;r_i\rangle$ is a disk in the plane defined as $\langle P;r\rangle = \ x\in R^2 \left\| x-P \right| \le r, P\in R^2, r\in R^+$. $N_{i,p}(t)$ is the p-degree B-Spline basis function.

A DBSC can be viewed as two parts: the center curve $P(t)=\sum_{i=0}^{n} N_{i,p}(t)P_i$, which is a B-Spline curve and the radius function $r(t)=\sum_{i=0}^{n} N_{i,p}(t)r_i$, which is a B-Spline scalar function. Most of the properties and algorithms of DBSC can be obtained by applying B-Spline curve and function to the two parts of DBSC respectively [3].

Extending principle

Given a cubic disk B-Spline curve $\langle D\rangle(t)=\sum_{i=0}^{n}N_{i,3}(t)\langle P_i;r_i\rangle$ with knot vector $T=[0,0,0,0,t_4,\ldots,t_n,1,1,1,1]$. $\langle Q;R\rangle$ is the given extending disk at point Q with radius R. Extending curve segment is expressed in cubic disk Bezier form $\langle B\rangle(u)=\sum_{i=0}^{3}B_i^3(u)\langle Q_i;R_i\rangle$ [1], where $B_i^3(u)$ is the cubic Bernstein basis function over $[0,1]$.

G²-connection between the original DBSC $\langle D\rangle(t)$ and the extending disk Bezier curve $\langle B\rangle(u)$ requires both the center curves and the boundaries of the regions to be G²-continuous. Here we discuss the G²-continuity conditions for the center curves and radius functions separately. Applying the G²-continuity conditions of B-Spline curves [5], the G²-continuity conditions for the center curves and radius functions of disk curves are illustrated in Eqn. (2.1) and Eqn. (2.2).

$$\begin{cases} Q(0)=P(1) \\ Q'(0)=\alpha_1 P'(1) \\ Q''(0)=\alpha_1^2 P''(1)+\beta_1 P'(1) \end{cases} \quad (2.1) \qquad \begin{cases} R(0)=r(1) \\ R'(0)=\alpha_2 r'(1) \\ R''(0)=\alpha_2^2 r''(1)+\beta_2 r'(1) \end{cases} \quad (2.2)$$

where $\alpha_1>0$, $\alpha_2>0$, and β_1, β_2 are arbitrary real numbers.

To make a simple solution, we make $\beta_1=0$ and $\beta_2=0$, and leave degree of freedom α_1 for center curve adjustment and degree of freedom α_2 for radius function adjustment. We expand Eqn. (2.1) and Eqn. (2.2). The control disks $\langle Q_i;R_i\rangle$ $i=0,1,2,3$ of the extending disk Bezier curve segment $\langle B\rangle(u)$ can be expressed.

We choose the exact energy variation [5] as the objective function for the fairness of the center curve of the extending disk Bezier curve: $E_{curve}=\int k^2(s)ds$, where ds is differential of curve arc length and $k(s)$ is curvature defined as $k(s)=\|Q'(u)\times Q''(u)\|/\|Q'(u)\|^3$. Through minimizing E_{curve}, α_1 can be determined. As for the degree of freedom α_2, as R must be positive, the constraints of α_2 can be calculated. For simplicity, the second order energy [2]: $E_{radius}=\int_0^1\|R''(u)\|^2du$, is used to measure the fairness of the radius function of the extending disk Bezier curve. α_2 can be determined through minimizing E_{radius}. Once the optimal α_1 and α_2 are determined, the extending disk Bezier curve segment can be achieved. The unclamped knot vector for the extended DBSC can be determined. The whole extended DBSC can be represented by re-computing the control disks through DBSC unclamping algorithm [4].

The experiment results of extending DBSC to one disk are shown in Fig. 1. Fig. 1(b) shows the extension result by accumulated chord length parameterization method in [4]. Fig. 1(c) shows the extension result by our minimal energy method. We can identify the effectiveness of our method through visual and numeric comparisons of the two methods. Also our proposed method applies for extension cases with multiple extending disks.

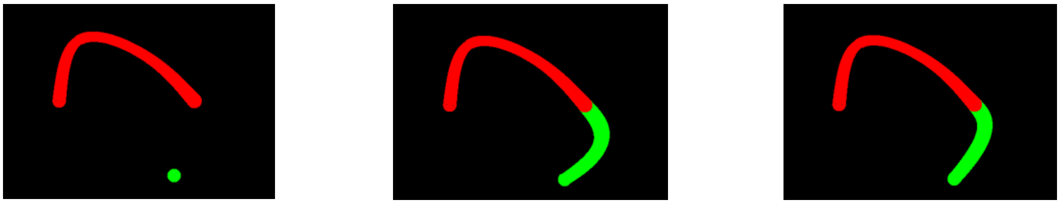

Fig. 1: Extension results of DBSC to one target disk. (a) The given DBSC and the target disk; (b)

Extension by accumulated chord length parameterization method; (c) Extension by minimal energy method.

Conclusions:

In this paper an extension algorithm for disk B-Spline curve with G^2 continuity is presented. The extending segment is expressed in disk Bezier curve and a whole extended disk B-Spline curve is represented. G^2-continuity is used to describe the smoothness of the joint disk and the extending disk Bezier curve fairness is achieved by minimizing energy objectives for the center curve and radius function respectively. The experimental results verify the effectiveness of our method. This work can lead to wider and further applications of DBSC in 2D region modeling.

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