Title:
Wavelet based Surface Decomposition with Boundary Continuity

Authors:
Aizeng Wang, aizwang@126.com, Beihang University, Beijing, P.R. China
Gang Zhao, zhaog@buaa.edu.cn, Beihang University, Beijing, P.R. China

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Introduction:

B-spline surfaces are a powerful mathematical representation of freeform surfaces and also play a primary role in computer-aided design (CAD) and computer aided manufacturing (CAM), especially in the area of industrial design such as ships, aircrafts and cars [1]. In addition, surface decomposition operations are often used in CAD systems to get surface simplification and fairing.

B-splines that provide a unified geometry representation of conic sections and free-form shapes have been widely used in CAD and CAM. However, for B-spline surfaces, all control points must lie topologically in a rectangular grid, and this makes they do not allow local refinement. To overcome these drawbacks, T-splines that are defined on a T-mesh and allow T-junctions in their control grid were proposed [2][3] (see Fig. 1). A T-junction is a control point which terminates a partial row or column in the control grid. Unlike B-spline surfaces, T-splines allow local refinement, and have good properties in surface merging and model simplification [2][3].

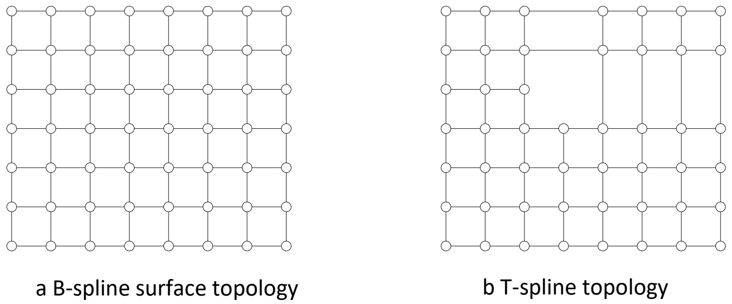

Fig. 1: B-spline surface and T-spline.

Wavelet analysis was introduced in 1980's, and it was widely utilized in the field of signal processing, computational geometry, and data compression due to its filtering property. The wavelet transform is a relatively new mathematical tool. In recent years, wavelet technology has been applied into the field of computer aided geometric design (CAGD) [4]. Later, B-spline wavelets were proposed and the relevant decomposition and reconstruction algorithms were given for curves and surfaces, especially in model simplification and fairing [5][6]. In the field of CAGD, B-spline wavelets are also used in the editing of curves and surfaces [7]. The surface wavelet decomposition aims to eliminate undesired features from the surface's shape. For the traditional method, an original surface can be decomposed into a scale part and a wavelet part by wavelet transform. However, there is a problem for the surface wavelet decomposition, that is the original surface and its scale part cannot preserve the boundary continuity. In the early research, Cho proposed a method for fairing the surfaces while preserving the boundary continuity [5]. However, this method can not preserve the boundary continuity with adjacent surfaces via wavelet transform directly. The boundary problem has affected the surface wavelet decomposition for many years and is still an open research area. Aiming at this problem, in this paper we will discuss how to preserve the boundary continuity for wavelet-based surface decomposition.

This paper presents a new decomposition algorithm for B-spline surfaces with boundary continuity. First, a B-spline surface is divided into a boundary part and a non-boundary part, and then the non-boundary one is decomposed into a scale part and a wavelet part; finally, the T-spline is utilized to reconstruct the boundary one and the scale one. In the new decomposition approach, the result surface is a T-spline which can preserve the boundary continuity.

Main idea:

The algorithm is summarized into the following three steps.

(Step 1) According to the corresponding conditions for certain boundary continuity, the original surface is divided into two parts: a boundary part and a non-boundary part;

(Step 2) Decompose the non-boundary part of the original surface into a scale part and a wavelet part;

(Step 3) Reconstruct a new surface by the scale part and the boundary part.

The final result of the reconstructed surface in Step 3 is a T-spline surface, which makes the boundary continuity with adjacent surfaces is automatically preserved in this algorithm. The main advantage of the new algorithm is that it preserves the boundary continuity which can not be preserved in the traditional surface wavelet decomposition.

Discussion: In our surface decomposition algorithm, the T-spline is used to preserve the boundary continuity. First, select the scale part, the wavelet part and the boundary part corresponding to the original surface, then reconstruct a new surface by synthesizing the scale part and the boundary part. Compared to the result obtained from the traditional surface wavelet decomposition method, our algorithm could preserve the boundary features by reconstructing a T-spline.

Conclusions:

This paper describes a novel wavelet-based decomposition algorithm for B-spline surfaces with boundary continuity. First, three parts are obtained from a surface, that is the scale part, the wavelet part and the boundary part; and then a T-spline by reconstructing the scale part and the boundary part is utilized to preserve the boundary continuity in the new surface decomposition approach. Our algorithm is accurate, automatic, efficient, and can be extensively applied to any B-spline surface model in computer aided design and computation geometry.

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