Title:
Design for Manufacturing of IFS Fractals from the Perspective of Barnsley's Fern-leaf

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Introduction:
Fractal geometry is effective in creating CAD models of complex shapes including the natural ones (tree, leaf, landscape) [1-6]. Here, a (complex) shape modeled by fractal geometry is referred to as fractal. One of the popular fractals is known as Barnsley's fern-leaf [2] that is created using the Iterative Function System (IFS), as follows:

$$
\begin{aligned}
&1: \textit{define } NI\,(\textit{an integer}),\ x_0 = 0,\ y_0 = 0\\
&2: \quad \textit{for } i = 1,\ldots,NI\\
&3: \quad\quad \textit{randomly assign } p_i \leftarrow [0,1]\\
&4: \quad\quad\quad \textit{if} \quad p_i \in [0,0.01) \quad \textit{then } x_i = 0,\ y_i = 1.6\,y_{i-1}\\
&5: \quad\quad\quad \textit{else if } p_i \in [0.01,0.86) \quad \textit{then } x_i = 0.85\,x_{i-1} + 0.04\,y_{i-1},\ y_i = -0.04\,x_{i-1} + 0.85\,y_{i-1} + 1.6\\
&6: \quad\quad\quad \textit{else if } p_i \in [0.86,0.93) \quad \textit{then } x_i = 0.23\,x_{i-1} - 0.26\,y_{i-1},\ y_i = 0.23\,x_{i-1} + 0.22\,y_{i-1} + 1.6\\
&7: \quad\quad\quad \textit{else if } p_i \in [0.93,1] \quad \textit{then } x_i = -0.15\,x_{i-1} + 0.28\,y_{i-1},\ y_i = 0.26\,x_{i-1} + 0.24\,y_{i-1} + 0.44\\
&8: \quad\quad\quad \textit{end if}\\
&9: \quad \textit{end for}
\end{aligned}
\tag{1}
$$

As defined in equation (1), four affine maps create a set of points in a recurrent manner and model the fern-leaf. Figure 1 shows the CAD models and a physical model (produced by milling) of the fern-leaf. Additive manufacturing (RP technology) can also be used for producing the physical models [7], if needed.

It is evident from the result shown in Fig. 1 that design for manufacturing is required to achieving accurate physical models.

Main idea:
The objective of this article is to elucidating a methodology of design for manufacturing of IFS fractals putting emphasis on the Barnsely's fern-leaf. To achieve the objective the following issues are addressed:

- What is the basic shape that repeats?
- What are the roles of affine maps?
- What is the scaling factor and how to control it?
- How to control the levels?
- How to create non-crossing tool paths?
- How to create facets and control their population?

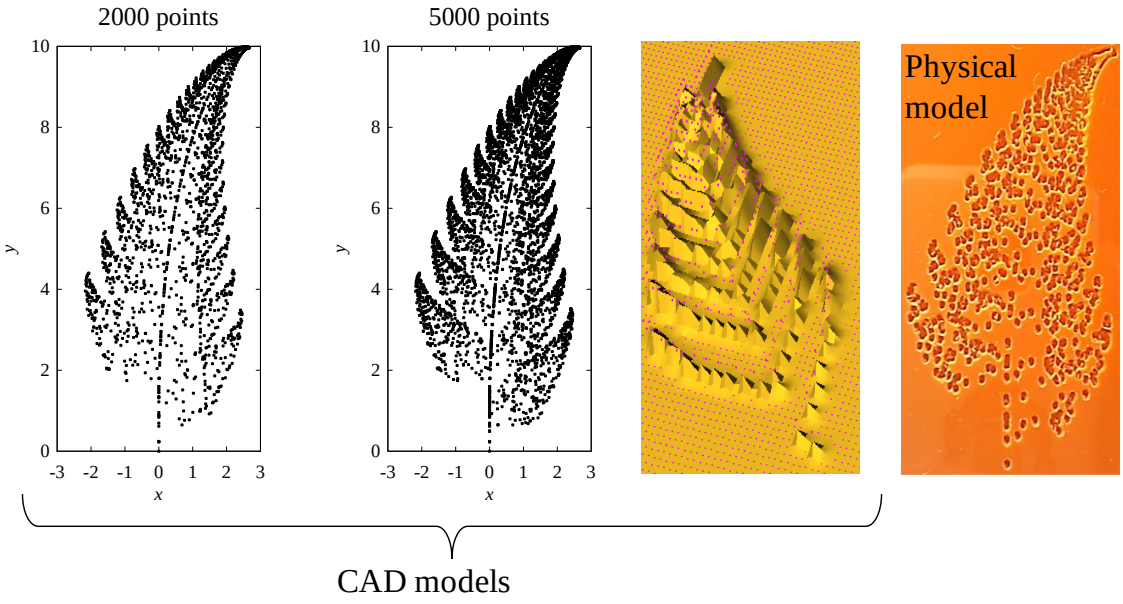

Fig. 1: CAD and Physical models of Barnsley's fern-leaf.

Figure 2 shows the results of CAD and physical models wherein the level of scaling has been controlled up to the third level and the non-crossing tool paths are created for CNC machining. Other results of physical models (3D printing) and the algorithms of controlling the occurrences of the affine maps shown in equation (1) will be described in the full paper.

Fig. 2: Results after remodeling the Barnsley's fern-leaf.

Conclusions:

It is possible to redesign Barnsley's fern-leaf in particular and IFS fractals in general without destroying their fractalness using some algorithms for controlling the roles of the underlying affine maps. It is also possible to create physical models of IFS fractals that resemble the CAD models. The measurement of self-similar dimension (e.g., fractal dimension) requires accurate fractals for calibration. The outcome of this study is useful in producing accurate physical models for calibrating the fractal dimension measuring instruments.

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