Title:
Parts Localization Oriented Practical Method for Point Projection on Model Surfaces based on Subdivision

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Introduction:
In algorithms of parts localization, the closest point of a measured point on model surface is chosen as its correspondence, which is also the key to process the localization of the measured data to CAD model. At present, Newton-type iterations are the most popular solving strategy. But, this process cannot provide full assurance that all solutions have been found [1]. Ma and Hewitt [2] have also showed that Newton-Raphson method occasionally still gives some wrong results even with a quite good initial value when applying it on the whole surface. Occasional errors maybe is trivial for the graphics processing, but it is fatal for some practical industrial applications. For example, in the machining allowance optimization, the distance between a point and its closest point on model surface is evaluated as the machining allowance, if wrong closest point is calculated at some area, an improper distribution of stock allowances, especially stock material shortage at this area, may appear such that the qualified blanks may be considered to be defective ones that need to be returned to the factory and is reworked, even although the CAD model actually can be enclosed within the blank [4]. For such industrial applications, the robustness of calculation of the closest point may be found to be more important and economical than the savings of the computing time. A different solving strategy, the methods based on subdivision [1, 2, 4], has been used for computing the closest point. When subdividing a surface, it is necessary to determine which segments contain the projection, which is the crux of the subdivision-based method. Different from those previous methods, the proposed method is to use the position relationships of the graph of the first derivative of the squared distance function and the u-v plane to discard the invalid segments. A simple formula is derived to facilitate the use of this new criterion.

Main idea:
For the sake of explanatory convenience, its basic principles are explained using Bézier surface. Mathematically, the point projection can be described as to find a corresponding point of a given point $\boldsymbol{p}_i$ on a surface $S(u,v)$ such that the distance between $\boldsymbol{p}_i$ and its corresponding point is minimal. The function to be minimized was

$$\min_{u,v}\left(d_p^S(u,v)\right)=\min_{u,v}(\|\boldsymbol{p}_i-S(u,v)\|^2) \tag{1}$$

If closest point is the inner point of the surface, the following condition is necessary, i.e. $\nabla d_p^S(u,v)=0$. Thus the closest point is turned into a problem of solving the roots of $\nabla d_p^S(u,v)=0$. In this paper, instead of using traditional numerical methods, a quadtree decomposition based method is given to solve it. An equivalent equation to $\nabla d_p^S(u,v)=0$ is given by

$$w(u,v)=\left(\frac{\partial d_p^S(u,v)}{\partial u}\right)^2+\left(\frac{\partial d_p^S(u,v)}{\partial v}\right)^2=0 \tag{2}$$

After substituting the first derivative of squared distance function $d_p^S(u,v)$ with respect to u and v, using the arithmetic for Bernstein polynomials referred to [5], $w(u,v)$ can be rewritten as a bivariate Bernstein-form polynomial as shown in Eq. (3)

$$w(u,v) = \sum_{i=0}^{4m}\sum_{j=0}^{4n} g_{i,j} B_{i,4m}(u) B_{i,4n}(v) \tag{3}$$

where $g_{i,j}$ are the Bernstein coefficients. The graph of $w(u,v)$ can be described by a Bézier surface over the $u-v$ plane, and it is called as the first derivative surface in this paper and is modeled by the following parametric equation

$$\begin{cases} S_w\colon \boldsymbol{w}(u,v) = \sum_{i=0}^{4m}\sum_{j=0}^{4n} \boldsymbol{g}_{i,j} B_{i,4m}(u) B_{i,4n}(v) \\ \boldsymbol{g}_{i,j} = \left[\frac{i}{4m}, \frac{j}{4n}, g_{i,j}\right]^{\mathrm{T}} \end{cases} \tag{4}$$

where $\boldsymbol{g}_{i,j}$ are the control points of the first derivative surface S_w. From Eq. (2), it can be seen that $w(u,v)$ is nonnegative, namely, no portion of S_w lies below the $u-v$ plane, which implies that if $w(u,v)$ is equal to zero, S_w has to be tangential to the $u-v$ plane and the tangent point is the closest point.

An adaptive quadtree decomposition on the $u-v$ parameter domain is adopted to narrow the region containing the tangent point, and de Casteljau algorithm is used to subdivide the $u-v$ domain into four sub-rectangular domains at the midpoints of u and v. In searching the closest point, S_w is subdivided recursively and control points of sub-$S_w^{(i)}$ are checked simultaneously. If all control points are completely above the $u-v$ plane, the node of the corresponding parameter domain is marked as one excluding the solutions. The searching process stops a depth d_t where the size of the domain is less than a predefined threshold ϵ_t, namely $2^{-d_t} \le \epsilon_t$. Then the quadtree is traversed and all unmarked nodes are collected at d_t from which the intervals $[u_l,u_h]\times[v_l,v_h]$containing possibly the closest point is produced and the closest point is calculated by $S((u_l+u_h)/2\,,(v_l+v_h)/2)$.

Using knot insertion technique, a B-spline surface can be easily subdivided into a set of Bézier surface. For each Bézier surface, the above algorithm is implemented to judge whether $w(u,v)=0$ hold or not. For Bézier surfaces satisfying this condition, quadtree decomposition is implemented to search the closest point. In such a way, the proposed method can be nicely generalized to B-spline surface. Since this method does not involve any iteration, it avoids the requirement of pro- viding a good initial value for achieving the proper result and the disadvantages of the traditional numerical methods.

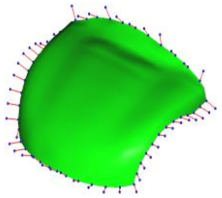

Fig. 1: Point projection on surface and boundary

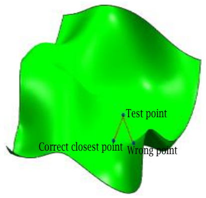

Fig. 2: Comparison between our method and Newton-Raphson method

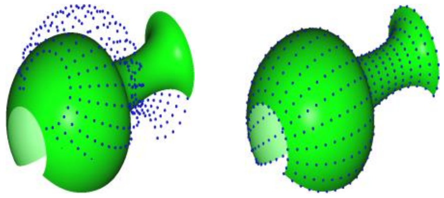

Fig. 3: Application in machining allowance optimization.

The proposed algorithm has been implemented on a PC in C++. The ability that the proposed method deals correctly with the point projection in any situations, especially for the points near the surface boundary has been demonstrated as shown in Fig. 1. For a point shown in Fig.2, when the subdivision interval is set as 10^{-3}, Newton-Raphson method produces a wrong answer, and our method lead to the proper projection. It has been also integrated into localization algorithm for machining allowance optimization and worked very well, as shown in Fig. 3.

Conclusions:

Experimental results demonstrate that the proposed method can provides full assurance that all solutions can be found for any conditions, especially for the boundary points. Moreover, it also avoids the requirements of providing a good initial value for achieving the proper result, and does not also produce the uncontrollable occasional error resulted from iteration thus it is nicely applicable to localization for machining allowance optimization and inspection of the machined parts based on CAD model.

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